International Journal of
Materials
& Structural Reliability

International Journal of Materials & Structural Reliability Vol.4, No.1, March 2006, 39-52

Reliability and Availability Evaluation of Large Renewal Systems

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Abstract

The paper proposes an approach to the solution of practically important problem of linking the asymptotic approach to reliability of large systems evaluation and the asymptotic approach to availability characteristics evaluation of renewal systems. This approach simplifies the traditional methods of availability investigation of large renewal systems and gives practically important in everyday usage tool for reliability and availability evaluation of the large renewal systems in the case when the system renovation time is either ignored or it is not ignored.

Firstly, the series-parallel multi-state system is considered and its reliability and risk characteristics are found. Next, asymptotic approach to reliability evaluation of the multi-state series-parallel system is introduced and linked with limit theorems of classical renewal theory. This join model is constructed for the system with ignored and non-ignored time of renovation. Finally, the asymptotic approach to reliability and availability evaluation of such a renewal system is applied to the port grain transportation system.

Keywords: Large renewal systems, Limit reliability function, Renewal theory application, Port transportation systems

1. Introduction

In the multi-state reliability analysis [1, 2] to define systems with degrading components we assume that all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$, the reliability states are ordered, the state 0 is the worst and the state z is the best and the component and the system reliability states degrade with time t without repair. The above assumptions mean that the states of the system with degrading components may be changed in time only from better to worse ones.

One of basic multi-state reliability structures with components degrading in time are series-parallel systems [3, 4]. To define them, we additionally assume that E_{ij} , $i = 1,2,\dots,k_n$, $j = 1,2,\dots,l_i$, k_n, $l_1, l_2, \dots, l_{k_n}$, $n \in N$, are components of a system, $T_{ij}(u)$, $i = 1,2,\dots,k_n$, $j = 1,2,\dots,l_i$, k_n, $l_1, l_2, \dots, l_{k_n}$, $n \in N$, are independent random variables representing the lifetimes of components E_{ij} in the state subset $\{u, u+1,..., z\}$ while they were in the state z at the moment t = 0, $e_{ij}(t)$ are components E_{ij} states at the moment t, $t \in (-\infty, \infty)$, while they were in the state z at the moment t = 0, $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u, u+1, ..., z\}$ while it was in the reliability state z at the moment $t = 0$ and $s(t)$ is the system reliability state at the moment t, $t \in (-\infty, \infty)$, given that it was in the state z at the moment $t = 0$.

Definition 1. A vector

$$R_{ij}(t,\cdot) = [R_{ij}(t,0), R_{ij}(t,1), \ldots, R_{ij}(t,z)],\ t \in (-\infty, \infty),$$

where

$R_{ij}(t,u) = P(e_{ij}(t) \geq u \mid e_{ij}(0) = z) = P(T_{ij}(u) > t)$, for
$t \in (-\infty, \infty)$, $u = 1,2,\ldots,z$, $i = 1,2,\ldots,k_n$, $j = 1,2,\ldots,l_i$, is the probability that the component E_{ij} is in the reliability state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in (-\infty, \infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a component E_{ij}.

Definition 2. A vector

$$\boldsymbol{R}_{k_n,l_n}(t,\cdot) = [\boldsymbol{R}_{k_n,l_n}(t,0), \boldsymbol{R}_{k_n,l_n}(t,1), \ldots,\ \boldsymbol{R}_{k_n,l_n}(t,z)],$$

where

$$\boldsymbol{R}_{k_n,l_n}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \quad \text{for } t \in (-\infty,\infty),\ u = 0,1,...,z,$$

is the probability that the system is in the reliability state subset $\{u, u+1, ..., z\}$ at the moment t, $t \in (-\infty, \infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a system.

It is clear that from *Definition 1* and *Definition 2*, for $u = 0$, we have $R_{ij}(t,0) = 1$ and $\boldsymbol{R}_{k_n,l_n}(t,0) = 1$.

Definition 3. A multi-state system is called series-parallel if its lifetime $T(u)$ in the state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = \max_{1 \leq i \leq k_n} \{ \min_{1 \leq j \leq l_i} \{T_{ij}(u)\} \},\ u = 1,2,\ldots,z.$$

Definition 4. A multi-state series-parallel system is called regular if $l_1 = l_2 = \cdots = l_{k_n} = l_n$, $l_n \in N$.

Definition 5. A multi-state regular series-parallel system is called non-homogeneous if it is composed of a, $1 \leq a \leq k_n$, $k_n \in N$, different types of series subsystems and the fraction of the ith type series subsystem is equal to q_i, where $q_i > 0$, $\sum_{i=1}^{a} q_i = 1$.

Moreover, the ith type series subsystem consists of e_i, $1 \leq e_i \leq l_n$, $l_n \in N$, types of components with reliability functions

$R^{(i,j)}(t,\cdot) = [1, R^{(i,j)}(t,1), \ldots, R^{(i,j)}(t,z)],$

where

$R^{(i,j)}(t,u) = 1 - F^{(i,j)}(t,u)$ for $t \in (-\infty, \infty)$, $j = 1,2,\ldots,e_i$, $u = 1,2,...,z,$

and the fraction of the jth type component in this subsystem is equal to p_{ij}, where $p_{ij} > 0$ and $\sum_{j=1}^{e_i} p_{ij} = 1.$

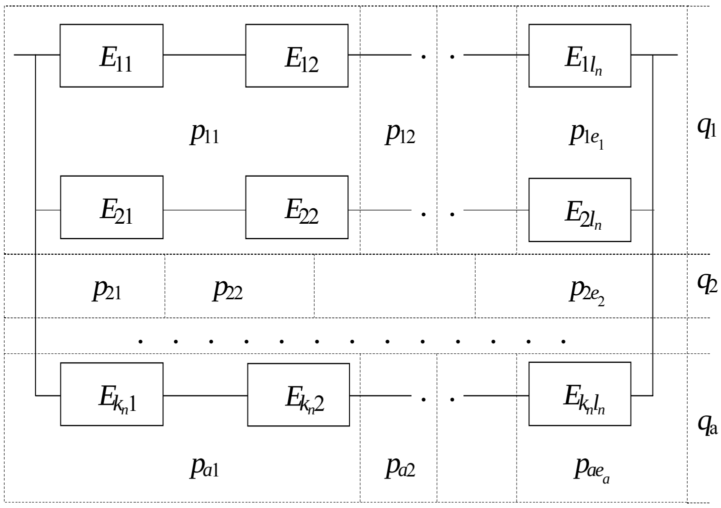

Fig. 1 The scheme of a regular non-homogeneous series-parallel system.

The reliability function of the multi-state non-homogeneous regular series-parallel system is given by

$$\boldsymbol{R}_{k_n,l_n}(t,\cdot) = [1, \boldsymbol{R}_{k_n,l_n}(t,1), \ldots, \boldsymbol{R}_{k_n,l_n}(t,z)], \quad (1)$$

where

$\boldsymbol{R}_{k_n,l_n}(t,u) = 1 - \prod_{i=1}^{a} [1 - [R^{(i)}(t,u)]^{l_n}]^{q_i k_n}$, for $t \in (-\infty, \infty)$, $u = 1,2,\ldots,z,$

and $R^{(i)}(t,u) = \prod_{j=1}^{e_i} [R^{(i,j)}(t,u)]^{p_{ij}}$, $i = 1,2,\ldots,a.$

Under these definitions, if $\boldsymbol{R}_{k_n,l_n}(t,u) = 1$ for $t \le 0$, $u = 1,2,\ldots,z,$ then

$$M(u) = \int_0^\infty \boldsymbol{R}_{k_n,l_n}(t,u)dt, \; u = 1,2,\ldots,z, \quad (2)$$

is the mean lifetime of the multi-state non-homogeneous regular series-parallel system in the reliability state subset $\{u, u+1, ..., z\}$, and the mean lifetime $\overline{M}(u),\ u = 1,2,\ldots,z,$ of this system in the state u can be determined from the following relationship

$$\overline{M}(u) = M(u) - M(u+1), \quad u = 1,2,..., z-1, \quad \overline{M}(z) = M(z). \tag{3}$$

Definition 6. A probability

$$r(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),\ t \in (-\infty, \infty),$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the reliability state z at the moment $t = 0$ is called a risk function of the multi-state non-homogeneous regular series-parallel system.

Considering *Definition 6* and *Definition 2*, we have

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_{k_n,l_n}(t, r), \quad t \in (-\infty, \infty), \tag{4}$$

and if τ is the moment when the system risk function exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{5}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

2. Large Multi-State Series-Parallel System

In the asymptotic approach to multi-state system reliability analysis [5] we are interested in the limit distributions of a standardized random variable $(T(u) - b_n(u))/a_n(u),\ u = 1,2,\ldots,z,$ where $T(u)$ is the lifetime of the multi-state non-homogeneous regular series-parallel system in the state subset $\{u, u+1, ..., z\}$ and $a_n(u) > 0,\ b_n(u) \in (-\infty, \infty),\ u = 1,2,\ldots,z,$ are some suitably chosen numbers, called normalizing constants.

Since

$$P((T(u) - b_n(u))/a_n(u) > t) = P(T(u) > a_n(u)t + b_n(u)) = \boldsymbol{R}_{k_n,l_n}(a_n(u)t + b_n(u), u),\ u = 1,2,\ldots,z,$$

then we assume the following definition.

Definition 7. A vector

$$\boldsymbol{\mathfrak{R}}(t, \cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1), \ldots, \boldsymbol{\mathfrak{R}}(t, z)],\ t \in (-\infty, \infty),$$

is called the limit multi-state reliability function of the multi-state non-homogeneous regular series-parallel system with reliability function $\boldsymbol{R}_{k_n,l_n}(t,\cdot)$ if there exist normalising constants $a_n(u) > 0,\ b_n(u) \in (-\infty, \infty),$ such that

$$\lim_{n\to\infty} \boldsymbol{R}_{k_n,l_n}(a_n(u)t + b_n(u), u) = \boldsymbol{\mathfrak{R}}(t, u) \quad \text{for } t \in C_{\boldsymbol{\mathfrak{R}}(u)},\ u = 1,2,...,z,$$

where $C_{\mathfrak{R}(u)}$ is the set of continuity points of $\mathfrak{R}(t,u)$.

The knowledge of the system limit reliability function allows us, for sufficiently large n and $t \in (-\infty,\infty)$, to apply the following approximate formula

$$\boldsymbol{R}_{k_n,l_n}(t,\cdot) = [1, \boldsymbol{R}_{k_n,l_n}(t,1), \dots, \boldsymbol{R}_{k_n,l_n}(t,z)] \cong [1, \mathfrak{R}(\frac{t-b_n(1)}{a_n(1)},1), \dots, \mathfrak{R}(\frac{t-b_n(z)}{a_n(z)},z)]. \qquad (6)$$

Proposition 1. If components of the non-homogeneous regular multi-state series-parallel system have exponential reliability functions

$$R^{(i,j)}(t,\cdot) = [1, R^{(i,j)}(t,1), \dots, R^{(i,j)}(t,z)],\ t \in (-\infty,\infty),$$

where

$$R^{(i,j)}(t,u) = 1 \quad \text{for } t < 0, \quad R^{(i,j)}(t,u) = \exp[-\lambda_{ij}(u)t] \quad \text{for } t \geq 0,$$

$$\lambda_{ij}(u) > 0,\ u = 1,2,\dots,z,\ i = 1,2,\dots,a,\ j = 1,2,\dots,e_i,$$

and $k_n \to k,\ l_n \to \infty,$

$$a_n(u) = \frac{1}{\lambda(u) l_n}, \quad b_n(u) = 0,$$

where $\lambda_i(u) = \sum_{j=1}^{e_i} \lambda_{ij}(u),\ \lambda(u) = \min_{1 \leq i \leq a}\{\lambda_i(u)\},$
then

$$\mathfrak{R}(t,\cdot) = [1, \mathfrak{R}(t,1), \dots, \mathfrak{R}(t,z)],$$

where

$$\mathfrak{R}(t,u) = 1 - \prod_{\{i:\lambda_i(u)=\lambda(u)\}} [1-\exp[-t]]^{q_i k} \quad \text{for } t \geq 0, \quad u = 1,2,...,z$$

is its limit reliability function.

From *Proposition 1* and after applying (6), we get the following approximate formula

$$\boldsymbol{R}_{k_n,l_n}(t,u) \cong 1 - \prod_{\{i:\lambda_i(u)=\lambda(u)\}} [1-\exp[-\lambda(u) l_n t]]^{q_i k} \quad \text{for } t \geq 0, \quad u = 1,2,...,z. \qquad (7)$$

3. Availability of Large Multi-State Series-Parallel System

First the asymptotic evaluation of elementary reliability characteristics of the renewal system with ignored time of renovation will be found. Next the renewal system with non-ignored time of renovation will be considered.

Proposition 2. If components of the non-homogeneous regular multi-state renewal series-parallel system with ignored time of renovation have exponential reliability functions

$$R^{(i,j)}(t,\cdot) = [1, R^{(i,j)}(t,1), \ldots, R^{(i,j)}(t,z)],\ t \in (-\infty,\infty),$$

where

$$R^{(i,j)}(t,u) = 1 \quad \text{for } t < 0, \quad R^{(i,j)}(t,u) = \exp[-\lambda_{ij}(u)t] \quad \text{for } t \geq 0,$$

$$\lambda_{ij}(u) > 0,\ u = 1,2,\ldots,z,\ i = 1,2,\ldots,a,\ j = 1,2,\ldots,e_i,$$

and $k_n \to k,\ l_n \to \infty,$

then:

i) the time $S_N(r)$ until the *Nth* exceeding of reliability critical state r of this system, for sufficiently large N, has approximately normal distribution $N(N\mu(r), \sqrt{N}\sigma(r))$, i.e.,

$$F^{(N)}(t,r) = P(S_N(r) < t) \cong F_{N(0,1)}\left(\frac{t - N\mu(r)}{\sqrt{N}\sigma(r)}\right), \text{ for } t \in (-\infty,\infty),\ r \in \{1,2,\ldots,z\},$$

ii) the expected value and the variance of the time $S_N(r)$ until the *Nth* exceeding the reliability critical state r of this system take respectively forms

$$E[S_N(r)] = N\mu(r), \quad D[S_N(r)] = N\sigma^2(r),$$

iii) the distribution of the number $N(t,r)$ of exceeding the reliability critical state r of this system up to the moment $t, t \geq 0$, for sufficiently large t, is approximately of the form

$$P(N(t,r) = N) \cong F_{N(0,1)}\left(\frac{N\mu(r) - t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}}\right) - F_{N(0,1)}\left(\frac{(N+1)\mu(r) - t}{\sigma(r)\sqrt{\frac{t}{\mu(r)}}}\right),$$

iv) the expected value and the variance of the number $N(t,r)$ of exceeding the reliability critical state r of this system at the moment $t, t \geq 0$, for sufficiently large t, approximately take respectively forms

$$H(t,r) = \frac{t}{\mu(r)}, \quad D(t,r) = \frac{t}{[\mu(r)]^3}[\sigma(r)]^2,$$

where

$$\mu(r) = \int_0^{+\infty} t\ d\tilde{F}(t,r), \quad [\sigma(r)]^2 = \int_0^{+\infty} t^2\ d\tilde{F}(t,r) - [\mu(r)]^2, \tag{8}$$

and

$$\tilde{F}(t,r) = 0 \quad \text{for} \quad t < 0, \quad \tilde{F}(t,r) = \prod_{\{i:\lambda_i(r)=\lambda(r)\}} [1 - \exp[-\lambda(r) l_n t]]^{q_i k} \quad \text{for} \quad t \geq 0,$$

$$\lambda_i(r) = \min_{1 \leq j \leq e_i} \{\lambda_{ij}(r)\}, \quad \lambda(r) = \max_{1 \leq i \leq a} \{\lambda_i(r)\}.$$

Proposition 3. If components of the non-homogeneous regular multi-state renewal series-parallel system with non-ignored time of renovation have exponential reliability functions

$$R^{(i,j)}(t,\cdot) = [1, R^{(i,j)}(t,1), \ldots, R^{(i,j)}(t,z)], \; t \in (-\infty, \infty),$$

where

$$R^{(i,j)}(t,u) = 1 \quad \text{for} \quad t < 0, \quad R^{(i,j)}(t,u) = \exp[-\lambda_{ij}(u) t] \quad \text{for} \quad t \geq 0,$$

$$\lambda_{ij}(u) > 0, \; u = 1,2,\ldots,z, \; i = 1,2,\ldots,a, \; j = 1,2,\ldots,e_i,$$

and $k_n \to k, \; l_n \to \infty,$

and random variables $Q^{(1)}(r), \; Q^{(2)}(r), ...,$ representing successive times of system renovations are independent and have an identical distribution function $\tilde{G}(t,r),$ with the expected value $E[Q(r)] = \mu_o(r)$ and the variance $D[Q(r)] = \sigma_o^2(r)$ then:

i) the time $\overline{\overline{S}}_N(r)$ until the *Nth* renovation of this system, for sufficiently large *N*, has approximately normal distribution

$N(N(\mu(r) + \mu_o(r)), \sqrt{N(\sigma^2(r) + \sigma_o^2(r))})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,r) = P(\overline{\overline{S}}_N(r) < t) \cong F_{N(0,1)}\left(\frac{t - N(\mu(r) + \mu_o(r))}{\sqrt{N(\sigma^2(r) + \sigma_o^2(r))}}\right), \quad \text{for}$$

$t \in (-\infty, \infty), \; N = 1,2,..., \; r \in \{1,2,...,z\},$

ii) the expected value and the variance of the time $\overline{\overline{S}}_N(r)$ until the *Nth* renovation of this system take respectively forms

$$E[\overline{\overline{S}}_N(r)] \cong N(\mu(r) + \mu_o(r)), \; D[\overline{\overline{S}}_N(r)] \cong N(\sigma^2(r) + \sigma_o^2(r)),$$

iii) the distribution function of the time $\overline{S}_N(r)$ until the *Nth* exceeding the reliability critical state *r* of this system takes form

$$\overline{F}^{(N)}(t,r) = P(\overline{S}_N(r) < t) \cong F_{N(0,1)}\left(\frac{t - N(\mu(r) + \mu_o(r)) + \mu_o(r)}{\sqrt{N(\sigma^2(r) + \sigma_o^2(r)) - \sigma_o^2(r)}}\right),$$

iv) the expected value and the variance of the time $\bar{S}_N(r)$ until the *Nth* exceeding the reliability critical state *r* of this system take respectively forms

$$E[\bar{S}_N(r)] \cong N\mu(r) + (N-1)\mu_o(r),\ D[\bar{S}_N(r)] \cong N\sigma^2(r) + (N-1)\sigma_o^2(r),$$

v) the distribution of the number $\bar{\bar{N}}(t,r)$ of the system renovations up to the moment $t, t \ge 0,$ is of the form

$$P(\bar{\bar{N}}(t,r) = N) \cong F_{N(0,1)}\Big(\frac{N(\mu(r)+\mu_o(r)) - t}{\sqrt{\frac{t}{\mu(r)+\mu_o(r)}(\sigma^2(r)+\sigma_o^2(r))}}\Big) - F_{N(0,1)}\Big(\frac{(N+1)(\mu(r)+\mu_o(r)) - t}{\sqrt{\frac{t}{\mu(r)+\mu_o(r)}(\sigma^2(r)+\sigma_o^2(r))}}\Big),$$

vi) the expected value and the variance of the number $\bar{\bar{N}}(t,r)$ of the system renovations up to the moment $t, t \ge 0,$ take respectively forms

$$\bar{\bar{H}}(t,r) \cong \frac{t}{\mu(r)+\mu_o(r)},\ \bar{\bar{D}}(t,r) \cong \frac{t}{[\mu(r)+\mu_o(r)]^3}(\sigma^2(r)+\sigma_o{}^2(r)),$$

vii) the distribution of the number $\bar{N}(t,r)$ of exceeding the reliability critical state *r* of this system up to the moment $t, t \ge 0,$ is of the form

$$P(\bar{N}(t,r) = N) \cong F_{N(0,1)}\Big(\frac{N(\mu(r)+\mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\Big) - F_{N(0,1)}\Big(\frac{(N+1)(\mu(r)+\mu_o(r)) - t - \mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu(r)+\mu_0(r)}(\sigma^2(r)+\sigma_o^2(r))}}\Big),$$

viii) the expected value and the variance of the number $\bar{N}(t,r)$ of exceeding the reliability critical state *r* of this system up to the moment $t, t \ge 0,$ are respectively given by

$$\bar{H}(t,r) \cong \frac{t+\mu_0(r)}{\mu(r)+\mu_o(r)},\ \bar{D}(t,r) \cong \frac{t+\mu_0(r)}{[\mu(r)+\mu_o(r)]^3}(\sigma^2(r)+\sigma_o{}^2(r)),$$

ix) the limiting availability coefficient of the system for sufficiently large *t*, is given by

$$K(t,r) \cong \frac{\mu(r)}{\mu(r)+\mu_o(r)},\quad t \ge 0,\ r \in \{1,2,...,z\},$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ for sufficiently large t, is given by the formula

$$K(t,\tau,r) \cong \frac{1}{\mu(r)+\mu_o(r)} \int_0^\infty \boldsymbol{R}_{k_n,l_n}(t,r)dt, \quad t \geq 0, \tau > 0,$$

where $\mu(r)$ and $\sigma(r)$ are given by formulae (8) and reliability function $\boldsymbol{R}_{k_n,l_n}(t,r)$ by (7).

4. Application

As an example we will analyse the reliability of the renewal port grain elevator with ignored and non-ignored time of renovation. This system is composed of four multi-state non-homogeneous series-parallel transportation subsystems and it is the basic structure in the Baltic Grain Terminal of the Port of Gdynia assigned to handle the clearing of exported and imported grain. One of the basic elevator functions is loading railway trucks with grain. The railway truck loading is performed in the following successive elevator operation steps [6]: gravitational passing of grain from the storage placed on the 8th elevator floor through 45 hall to horizontal conveyors placed in the elevator basement, transport of grain through horizontal conveyors to vertical bucket elevators transporting grain to the main distribution station placed on the 9th floor, gravitational dumping of grain through the main distribution station to the balance placed on the 6th floor, dumping weighed grain through the complex of flaps placed on the 4th floor to horizontal conveyors placed on the 2nd floor, dumping of grain from horizontal conveyors to worm conveyors, dumping of grain from worm conveyors to railway trucks.

In loading the railway trucks with grain the following elevator transportation subsystems take part: S_1 – horizontal conveyors of the first type, S_2 – vertical bucket elevators, S_3 – horizontal conveyors of the second type, S_4 – worm conveyors.

Taking into account the quality of work of the considered transportation system we distinguish the following three reliability states of its components [7]: state 2 – the state ensuring the largest quality of the conveyor work, state 1 – the state ensuring less quality of the conveyor work caused by throwing grain off the belt, state 0 – the state involving failure of the conveyor.
The structure of the port grain transportation system is given in Figure 2.

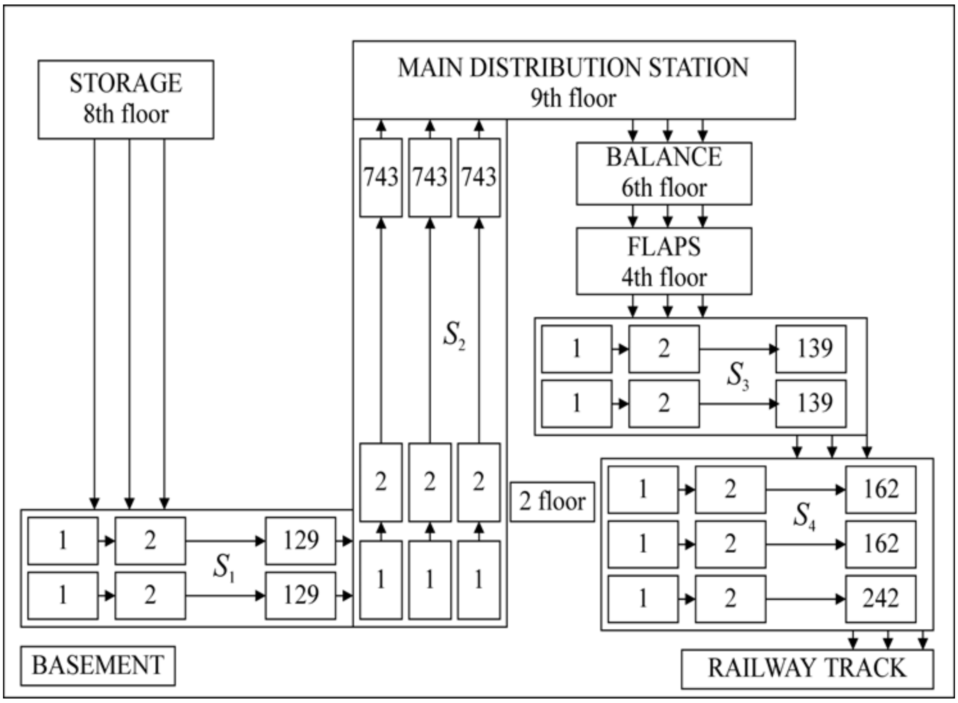

Fig. 2 The scheme of the grain transportation system.

Further we will discuss only the subsystem S_3, the remaining subsystems may be analyzed in the same way. It consists of $k_n = 2$ identical belt conveyors, each composed of $l_n = 139$ components. In each conveyor there is one belt with reliability functions

$$R^{(1,1)}(t,1) = \exp[-0.126t],\ R^{(1,1)}(t,2) = \exp[-0.167t],$$

two drums with reliability functions

$$R^{(1,2)}(t,1) = \exp[-0.0437t],\ R^{(1,2)}(t,2) = \exp[-0.048t],$$

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$$R^{(1,3)}(t,1) = \exp[-0.0798t],\ R^{(1,3)}(t,2) = \exp[-0.0978t]$$

and 19 supporting rollers with reliability functions

$$R^{(1,4)}(t,1) = \exp[-0.0714t],\ R^{(1,4)}(t,2) = \exp[-0.0798t] \quad \text{for} \quad t \geq 0.$$

The subsystem S_3 is a non-homogeneous regular series-parallel system with parameters

$$k_n = k = 2,\ l_n = 139,\ a = 1,\ q_1 = 1,\ e_1 = 4,$$

$$p_{11} = 1/139,\ p_{12} = 2/139,\ p_{13} = 117/139,\ p_{14} = 19/139,$$

$\lambda_{11}(1) = 0.126,\ \lambda_{12}(1) = 0.0437,\ \lambda_{13}(1) = 0.0798,\ \lambda_{14}(1) = 0.0714,$

$\lambda_{11}(2) = 0.167,\ \lambda_{12}(2) = 0.048,\ \lambda_{13}(2) = 0.0978,\ \lambda_{14}(2) = 0.0798.$

Since

$$\lambda_1(1) = \sum_{j=1}^{4} p_{1j}\lambda_{1j}(1) = \frac{1}{139}0.126 + \frac{2}{139}0.0437 + \frac{117}{139}0.0798 + \frac{19}{139}0.0714 = 0.0785,$$

$\lambda(1) = \min\{0.0785\} = 0.0785$,

$$\lambda_1(2) = \sum_{j=1}^{4} p_{1j}\lambda_{1j}(2) = \frac{1}{139}0.167 + \frac{2}{139}0.048 + \frac{117}{139}0.0978 + \frac{19}{139}0.0798 = 0.0951,$$

$\lambda(2) = \min\{0.0951\} = 0.0951$,

then applying *Proposition 1* with normalising constants

$$a_n(1) = \frac{1}{0.0785 \cdot 139} \cong \frac{1}{10.9115},\ b_n(1) = 0,$$

$$a_n(2) = \frac{1}{0.0951 \cdot 139} \cong \frac{1}{13.2189},\ b_n(2) = 0,$$

we conclude that

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1), \boldsymbol{\mathfrak{R}}(t,2)] = [1, 1-[1-\exp[-t]]^2, 1-[1-\exp[-t]]^2] \quad \text{for } t \geq 0$$

is the subsystem S_3 limit reliability function and from (7), we get that the reliability of the subsystem S_3 is given by

$$\boldsymbol{\mathfrak{R}}_{2,139}(t,\cdot) = [1, \boldsymbol{R}_{2,139}(t,1), \boldsymbol{R}_{2,139}(t,2)], \tag{9}$$

where

$$\boldsymbol{R}_{2,139}(t,1) = \boldsymbol{\mathfrak{R}}(10.9115t,1) = 1-[1-\exp[-10.9115t]]^2 \quad \text{for} \quad t \geq 0, \tag{10}$$

$$\boldsymbol{R}_{2,139}(t,2) = \boldsymbol{\mathfrak{R}}(13.2189t,2) = 1-[1-\exp[-13.2189t]]^2 \quad \text{for} \quad t \geq 0. \tag{11}$$

Hence we get that

$\mu(1) = 0.1375,\ \sigma(1) = 0.1122$ and $\mu(2) = 0.1135,\ \sigma(2) = 0.0926.$

Considering the port grain transportation system in the case when time of renovation is ignored and applying *Proposition 2* for obtained reliability function (9)-(11) at critical state $r = 2$ we get that:

i) the time $S_N(2)$ until the *Nth* exceeding the reliability critical state $r = 2$ of this system, for sufficiently large *N*, has approximately normal distribution $N(0.1135N, 0.0926\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t) \cong F_{N(0,1)}(\frac{t - 0.1135N}{0.0926\sqrt{N}}),\ t \in (-\infty, \infty),$$

ii) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding the reliability critical state $r = 2$ of this system take respectively forms

$$E[S_N(2)] = 0.1135N, \quad D[S_N(2)] = 0.0086N,$$

iii) the distribution of the number $N(t,2)$ of exceeding the reliability critical state $r = 2$ of this system up to the moment $t, t \geq 0,$ for sufficiently large *t*, is approximately of the form

$$P(N(t,2) = N) \cong F_{N(0,1)}(\frac{0.1135N - t}{0.0926\sqrt{8.8106t}}) - F_{N(0,1)}(\frac{0.1135(N+1) - t}{0.0926\sqrt{8.8106t}}),\ N = 0,1,2,\ldots,$$

iv) the expected value and the variance of the number $N(t,2)$ of exceeding the reliability critical state $r = 2$ of this system at the moment $t, t \geq 0,$ for sufficiently large *t*, approximately take respectively forms

$$H(t,2) = 8.8106t, \quad D(t,2) = 5.8645t.$$

Further, we assume that the grain elevator is repaired after its failure and that the time of the system renovation is not ignored and it has exponential distribution

$$\tilde{G}(t,r) = 1 - \exp[-200t] \text{ for } t \geq 0,$$

with the mean value and the standard deviation given respectively by

$$\mu_0 = 0.005 \text{ and } \sigma_0 = 0.005.$$

Thus, for the renewal system with non-ignored time of renovation from *Proposition 3* we obtain that:

i) the time $\overline{\overline{S}}_N(2)$ until the *Nth* renovation of this system, for sufficiently large *N*, has approximately normal distribution $N(0.1185N, 0.0929\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)}(t,2) = P(\overline{\overline{S}}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 0.1185N}{0.0929\sqrt{N}}), \text{ for } t \in (-\infty, \infty),\ N = 1,2,\ldots,$$

ii) the expected value and the variance of the time $\overline{\overline{S}}_N(2)$ until the *Nth* renovation of this system take respectively forms

$$E[\overline{\overline{S}}_N(2)] \cong 0.1185N, \quad D[\overline{\overline{S}}_N(2)] \cong 0.0086N,$$

iii) the distribution function of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state $r = 2$ of this system takes form

$$\overline{F}^{(N)}(t,2) = P(\overline{S}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 0.1185N + 0.005}{\sqrt{0.0086N + 0.000025}}),$$

iv) the expected value and the variance of the time $\overline{S}_N(2)$ until the *Nth* exceeding the reliability critical state $r = 2$ of this system take respectively forms

$$E[\overline{S}_N(2)] \cong 0.1135N + (N-1)0.005,\ D[\overline{S}_N(2)] \cong 0.0086N + (N-1)0.000025,$$

v) the distribution of the number $\overline{\overline{N}}(t,2)$ of the system renovations up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{\overline{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{0.1185N - t}{\sqrt{0.0726t}}) - F_{N(0,1)}(\frac{0.1185(N+1) - t}{\sqrt{0.0726t}}),$$

vi) the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of the system renovations up to the moment $t, t \geq 0,$ take respectively forms

$$\overline{\overline{H}}(t,2) \cong 8.4388t, \quad \overline{\overline{D}}(t,2) \cong 5.1682t,$$

vii) the distribution of the number $\overline{N}(t,2)$ of exceeding the reliability critical state $r = 2$ of this system up to the moment $t, t \geq 0,$ is of the form

$$P(\overline{N}(t,2) = N) \cong F_{N(0,1)}(\frac{0.1185N - t - 0.005}{\sqrt{0.0726(t + 0.005)}}) - F_{N(0,1)}(\frac{0.1185(N+1) - t - 0.005}{\sqrt{0.0726(t + 0.005)}}),$$

viii) the expected value and the variance of the number $\overline{N}(t,r)$ of exceeding the reliability critical state $r = 2$ of this system up to the moment $t, t \geq 0,$ are respectively given by

$$\overline{H}(t,2) \cong 8.4388(t + 0.005),\ \overline{D}(t,2) \cong 5.1682(t + 0.005),$$

ix) the limiting availability coefficient of the system for sufficiently large *t*, is given by

$$K(t,2) \cong 0.9578, \quad t \geq 0,$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ for sufficiently large *t*, is given by the formula

$$K(t,\tau,2) \cong 8.4388 \int_0^{\infty} [1 - [1 - \exp[-13.2189t]]^2]dt \cong 0.9578 \quad \text{for} \quad t \geq 0, \tau > 0.$$

If a critical reliability state of the system is $r = 2$, then from (4) the system risk function takes the form

$\boldsymbol{r}(t) \cong 1 - \boldsymbol{R}_{2,139}(t,2)$,

where $\boldsymbol{R}_{2,139}(t,2)$ is given by (11).

Hence, from (5), the moment when the risk exceeds the critical level $\delta = 0.05$ is $\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.02$.

Conclusions

In the paper new theorems joining limit theorems on order statistics and limit theorems from renewal theory are proposed. Methods of their application in reliability and availability characteristics of large renewal systems in the case when the system renovation time is ignored and it is not ignored are proposed and illustrated with the series-parallel multi-state system.

Application of the proposed methods is illustrated in the reliability and availability evaluation of the port grain transportation system. The reliability data concerned with the operation process and reliability functions of the port grain transportation system components are not precise. They are coming from experts and are concerned with the mean lifetimes of the system components and with the conditional sojourn times of the system under arbitrary assumption that their distributions are exponential.

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